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PALINDROMES AND TWO-DIMENSIONAL STURMIAN SEQUENCES

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ABSTRACT

This paper introduces a two-dimensional notion of palindrome for rectangular factors of double sequences: these palindromes are defined as centrosymmetric factors. This notion provides a characterization of two-dimensional Sturmian sequences in terms of two-dimensional palindromes, generalizing to double sequences the results in [13].

Keywords: palindromes, double sequences, generalized Sturmian sequences, symbolic dynamics, combinatorics on words.

1. Introduction

This paper studies some properties of symmetry for the rectangular factors of a family of two-dimensional sequences obtained as a binary coding of a $\mathbb{Z}^2$-action defined on the one-dimensional torus $\mathbb{T}^1(=\mathbb{R}/\mathbb{Z})$ by two irrational rotations. More precisely, such a sequence $(U_{m,n})_{(m,n)\in\mathbb{Z}^2}$ is defined on the alphabet $\{0,1\}$ as follows: consider a partition of the unit circle into two half-open intervals I_0 and I_1; let $\alpha,\beta,\gamma\in\mathbb{R}$ with $\alpha\notin\mathbb{Q}$; we have

$$\forall (m,n)\in\mathbb{Z}^2,\ (U_{m,n}=0 \iff m\alpha+n\beta+\gamma\in I_0 \text{ modulo } 1).$$

We will consider in particular the case where I_0 has length α and $1,\alpha,\beta$ are rationally independent. Such sequences can be considered as a generalization of Sturmian sequences. Recall that Sturmian sequences code the approximation of a line by a discrete line made of horizontal and vertical segments with integer vertices (see for instance [8] and the surveys [9, 17]). This two-dimensional generalization of Sturmian sequences has been introduced in [23, 6] and is closely connected (via a letter-to-letter