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GRAPHS WITH AUTOMATIC PRESENTATIONS OVER A UNARY ALPHABET [1]

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ABSTRACT

A relational structure is *automatic* if its domain and atomic relations are recognized by finite automata (FA). In this paper, a finite automaton recognizable n-ary relation is one accepted by a synchronous n-tape finite automaton. A structure is *automatically presentable* if it is isomorphic to an automatic structure. We focus on the class of automatic graphs whose domains are unary strings. The main result characterizes the isomorphism types of these graphs using a graph-theoretic construction called an *unwinding*.

Keywords: automatic graphs, automatic presentations, automatic structures, computable presentations, synchronous n-tape automata.

1. Introduction

We investigate the relationship between finite automata and relational structures. A structure is called *automatic* if its domain and atomic relations can be computed by finite automata. For instance, a linear order $\mathcal{L} = (L, \leq^L)$ is automatic over an alphabet Σ if both the domain $L \subset \Sigma^\star$ and the ordering $\leq^L \subset \Sigma^{\star 2}$ are finite automaton (FA) recognizable. Historically, the use of finite automata as a tool for studying computational and logical properties of structures dates to the 1960's with the works of BÜCHI [3] and ELGOT [9]. Their approaches initiated the use of finite automata to solve decision problems. See THOMAS [25] for a survey paper on the use of automata in the framework of mathematical logic. JOHNSON [17] investigates classes related to

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