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LIMITED IL SYSTEMS

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ABSTRACT

In this paper, the limitation process of k-limited and uniformly k-limited 0L systems is transferred to IL systems, that is to Lindenmayer systems with interaction. For non-extended systems, according to the different limitations of the systems, we get incomparability results of the corresponding language families. According to the number of left and right neighbours which a cell (symbol) is allowed to interact with, different inclusion and incomparability results are given. For extended systems, one neighbour to the right or to the left is enough to generate all recursively enumerable languages. Propagating such systems generate all ε-free context-sensitive languages.

Keywords: Formal languages, Lindenmayer systems, IL systems, limited 0L systems

1. Introduction

In [8] and [9], *A. Lindenmayer* introduced developmental systems with interactions to model the development of filamentous multicellular organisms where the different cells are allowed to interact with each other. Later on, these systems were called IL systems where the letter 'I' stands for interaction and the letter 'L' for Lindenmayer. Text book representations of IL systems can be found in [7, Chapters 6 and 7.2] and in [11, Chapter VI.1]. An (m,n)IL system is a system such that the substitution of a cell depends on its m left-sided and on its n right-sided neighbours. A special case of the IL systems are the $(0,0)$IL systems, called 0L systems, which have been investigated intensively (e. g., see [7, 11]). Since the fully parallel rewriting of 0L system is not always realistic, limitations of the rewriting process have been introduced in [13] and [14]. For a natural number k, in every derivation step of a k-limited 0L system [13], k occurrences of every symbol of the system's alphabet are rewritten in the word considered (if present else correspondingly less), for uniformly k-limited 0L systems [14] this are k arbitrary symbols of the word. These systems have been investigated in quite a lot of further papers (e. g., see [1, 2, 4, 5, 6, 15, 16, 17]). E. g., it could be proved that the family $\mathscr{L}$(ET0L) of ET0L languages is strictly included in the family $\mathscr{L}$(klET0L) of k-limited ET0L languages (see [13]) and that there even exist non-recursive k-limited ET0L languages (see [4] and [5]).