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## ON THE TRACE PRODUCT AND SOME FAMILIES OF LANGUAGES CLOSED UNDER PARTIAL COMMUTATIONS

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## ABSTRACT

New automata-theoretic constructions for the product of recognizable trace languages are given. This study leads to investigate the effect of partial (and total) commutations on some families of recognizable languages (the Straubing-Thérien's hierarchy of starfree languages).

Keywords: Trace languages, finite automata, trace product, formal languages

## 1. Introduction

In this paper we consider some problems related to the product of trace languages. It is well known that the product of recognizable trace languages is still recognizable. There exist several different proofs of such a result that use algebraic or combinatorial arguments (see Section 5 for references). However, no very much is known about the trace product for languages that are not recognizable.

The initial motivation of our work was to investigate new automata-theoretic constructions for the trace product, that we consider relevant both to theoretical questions and to some applications.

The starting idea of our approach, which takes suggestions from classical algorithms converting regular expressions into finite automata, is to regard the different occurrences of the same letter in distinct factors of the product as different symbols. So, given a product  $X_1X_2...X_n$  of n trace languages over the alphabet  $\Sigma$ , we introduce

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