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GRÖBNER BASES AND THE DEFINING POLYNOMIAL OF A CONTEXT-FREE GRAMMAR GENERATING FUNCTION

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ABSTRACT

We consider proper algebraic systems as defined in [7] and reprove via Gröbner bases algorithms that the quasiregular solution of such a system is algebraic. In this context, the effective primary decomposition of a polynomial ideal resp. the effective decomposition of an affine algebraic variety into irreducible components are alternatives to the Kuich-Salomaa elimination algorithm described in [7]. Both here applied decompositions are based on the construction of a Gröbner basis of an elimination ideal via Buchberger's algorithm. We reprove then in a constructive way that the generating function of each nonterminal of a context-free grammar is algebraic and also that the generating function of a language, generated by an unambiguous context-free grammar is algebraic (Chomsky-Schützenberger [4]).

Keywords: Gröbner bases, primary ideal decomposition, context-free grammar generating function, algebraic systems

1. Introduction

It is well known that the generating function of each nonterminal of a context-free grammar is algebraic. An effective method to show this fact by constructing the defining polynomial of this generating function is given by the Kuich-Salomaa algorithm [7]. For the construction of this defining polynomial it is there used elimination theory via systems of resultants, where further operations like Liouville substitutions and affine transformations are applied. This results in a rather involved algorithm that is unfortunately not very well suited to an implementation on a computer algebra system.

As described in this note, the whole elimination process can also be established via algorithms that use Gröbner bases. Due to the importance of Gröbner bases in many mathematical branches the Buchberger-algorithm for generating Gröbner bases as well as many ideal operations that use Gröbner bases are well implemented on many computer algebra systems. Thus this approach might have the advantage of an easier effective construction of the defining polynomial, which can then be used e. g. for decision procedures (see [7, 9]).