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ON THE LENGTH OF THE WADGE HIERARCHY OF ω-CONTEXT-FREE LANGUAGES [1]

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ABSTRACT

We prove in this paper that the length of the Wadge hierarchy of ω-context-free languages is greater than the Cantor ordinal ε_ω, which is the ω^{th} fixed point of the ordinal exponentiation of base ω. We show also that there exist some $\mathbf{\Sigma}^0_\omega$-complete ω-context-free languages, improving previous results on ω-context-free languages and the Borel hierarchy.

Keywords: ω-context-free languages; infinitary context-free languages; topological properties; Borel hierarchy; Wadge hierarchy; conciliating Wadge hierarchy

1. Introduction

In the sixties Büchi studied the ω-languages accepted by finite automata to prove the decidability of the monadic second order theory of one successor over the integers. Since then the so called ω-regular languages have been intensively studied, see [43, 31] for many results and references. The extension to ω-languages accepted by pushdown automata has also been investigated, firstly by Cohen and Gold, Linna, Nivat, see Staiger's paper [42] for a survey of this work, including acceptance of infinite words by more powerful accepting devices, like Turing machines. A way to investigate the complexity of ω-languages is to consider their topological complexity. Mc Naugthon's Theorem implies that ω-regular languages are boolean combinations of $\mathbf{\Pi}^0_2$-sets. We proved that ω-context-free languages (accepted by pushdown automata with a Büchi or Muller acceptance condition) exhaust the finite ranks of the Borel hierarchy, [16], that there exist some ω-context-free languages (ω-CFL) which are analytic but non Borel sets, [21], and that there exist also some ω-CFL which are Borel sets of infinite rank, [22].

On the other hand the Wadge hierarchy of Borel sets is a great refinement of the Borel hierarchy and it induces on ω-regular languages the now called Wagner

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