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INTERVALS OF PARTIAL CLONES CONTAINING MAXIMAL CLONES [1]

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ABSTRACT

Let $k \geq 2$ and $\mathbf{k}$ be a k-element set. Denote $\mathrm{Op}(\mathbf{k})$ the set of all total functions on $\mathbf{k}$. We study the set $\widehat{A}$ of all partial clones C on $\mathbf{k}$ whose total component $C \cap \mathrm{Op}(\mathbf{k})$ is a given maximal clone A. First we recall and establish some general facts, then we completely describe the set $\widehat{A}$ where A is a maximal clone determined by a central or an equivalence relation on $\mathbf{k}$. Furthermore we study a subset of $\widehat{A}$ for which A is a maximal clone determined by a bounded order. Here the problem turns out to be quite complex. Finally, we show that $\widehat{A}$ is finite whenever A is a maximal clone determined by a fixed-point-free permutation consisting of cycles of same length p, where p is a prime divisor of k. We also give a complete description of $\widehat{A}$ in the cases $p = 2, 3$.

Keywords: Partial clones, maximal clones, intervals of partial clones

1. Introduction

Let $k \geq 2$ and $\mathbf{k}$ be a k-element set. Denote by $\mathrm{Par}(\mathbf{k})$ the set of all partial functions on $\mathbf{k}$ and let $\mathrm{Op}(\mathbf{k})$ be the set of all total functions on $\mathbf{k}$, that is $\mathrm{Op}(\mathbf{k})$ consists of all everywhere defined functions on $\mathbf{k}$. A *partial clone* on $\mathbf{k}$ is a subset of $\mathrm{Par}(\mathbf{k})$ closed under composition and containing all the projections on $\mathbf{k}$. A partial clone contained

[1] To memory of Professor Dietmar Schweigert (1940 – 2006)