

LUKASIEWICZ LOGIC AND WEIGHTED LOGICS OVER MV-SEMRINGS¹

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ABSTRACT

We connect Łukasiewicz logic, a well-established many-valued logic, with weighted logics, recently introduced by Droste and Gastin. We use this connection to show that for formal power series with coefficients in semirings derived from standard MV-algebras, recognizability and definability in a fragment of second order Łukasiewicz logic coincide. This implies decidability properties for these fragments of Łukasiewicz logic.

Keywords: Weighted automata, weighted logic, Łukasiewicz logic, MV-algebras

1. Introduction

Recently, Droste and Gastin introduced *weighted logics* in [7]. In weighted logics, formulas are interpreted in semirings. The connectives \vee and \wedge reflect the semiring operations. Since semirings do not provide a truth function for negation, in weighted logics, negation is restricted to atomic formulas. Weighted logics can be viewed as many-valued second order logics on finite words and related structures (like trees, traces, pictures). In [7], a suitable fragment of it was shown to be expressively equivalent, for any commutative semiring, to the semantics of weighted automata.

In this paper, we compare weighted logics to traditional many-valued logics, especially Łukasiewicz logic. Łukasiewicz logic emerged in 1920 as three-valued logic and was soon extended to the infinite set of truth values $[0, 1]$. Like other many-valued logics, Łukasiewicz logic was developed as a generalization of two-valued logic extending the set of truth values while keeping as many as possible intuitive properties of the classical connectives [11, 12, 13]. In addition to the classical connectives \vee , \wedge and \neg , Łukasiewicz logic contains two connectives $\underline{\vee}$ (strong disjunction) and $\&$ (strong conjunction). Formulas of Łukasiewicz logic are interpreted in the standard MV-algebra $([0, 1], \oplus, \otimes, \neg, 0, 1)$ where $\neg x = 1 - x$ for all $x \in [0, 1]$, the truth function of $\underline{\vee}$ is the Łukasiewicz t -conorm \oplus and the truth function of $\&$ is the Łukasiewicz t -norm \otimes .

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