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ON REPRESENTABLE GRAPHS

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ABSTRACT

A graph $G = (V, E)$ is representable if there exists a word W over the alphabet V such that letters x and y alternate in W if and only if $(x, y) \in E$ for each $x \neq y$. If W is k-uniform (each letter of W occurs exactly k times in it) then G is called k-representable. We prove that a graph is representable if and only if it is k-representable for some k. Examples of non-representable graphs are found in this paper. Some wide classes of graphs are proven to be 2- and 3-representable. Several open problems are stated.

Keywords: Combinatorics on words, representation, (outer)planar graphs, prisms, Perkins semigroup, graph subdivisions

1. Introduction

To the nodes of a graph G, we assign distinct letters from some alphabet. A graph G is representable if there exists a word W such that any two letters, say x and y, alternate in W if and only if G contains an edge between the nodes corresponding to x and y. In such a situation we say that W represents G.

Representable (in our sense) graphs are considered in [1] to obtain asymptotic bounds on the free spectrum of the widely-studied *Perkins semigroup*, $\mathbf{B}_2^1$, which has played central role in semigroup theory since 1960, particularly as a source of examples and counterexamples. Recall that the Perkins semigroup has the elements

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

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