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GRAPH REPRESENTATION FUNCTIONS COMPUTABLE BY FINITE AUTOMATA

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ABSTRACT

We consider a simple model for representing a graph in computer memory in which every vertex is assigned a word in a finite alphabet - *vertex code* - and the adjacency of two vertices is a function Ψ of their codes. The function Ψ is called the *representation function*. We say that Ψ is *universal* if a Ψ-representation exists for every simple graph G. In this paper, we study representation functions computable by automata with two states. The main result is a criterion characterizing the universal functions. In the case of binary alphabet, we provide some bounds on the dimension of minimum Ψ-representation.

Keywords: Vertex representation, graph coding, finite automata

1. Introduction

This paper concerns the problem of representing graphs in computer memory. We study *vertex representation*, i. e., the approach in which every vertex is assigned a word in a finite alphabet - *the code of the vertex* - and the adjacency of two vertices is a function Ψ of their codes. This approach is useful in distributed computing when the graph does need to be stored in one place. But whenever a processor needs to determine adjacency, it can do so only by looking at the codes of the vertices. Several important representation models fall in this framework.

The function Ψ associated with vertex representation is called the *representation function*. To make the computations simple and fast we study functions computable by finite automata. We say that Ψ is a *universal* function if a Ψ-representation exists for every simple graph G. The main result is a criterion characterizing the universal functions computable by automata with two states. In the case of binary alphabet, we provide some bounds on the dimension of minimum Ψ-representation.

The paper is organized as follows. In the rest of this section, we give basic definitions and notations. In Section 2, we formally define the notion of vertex representation and mention several important representation models of this type. Section 3

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