

## ON THE GROWTH OF CONTEXT-FREE LANGUAGES

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### ABSTRACT

This paper concerns some algebraic and algorithmical problems on the growth function of formal languages. In particular, we establish a regularity condition for sparse context-free languages.

*Keywords:* Growth function, bounded language, context-free language

### 1. Introduction

This paper mainly concerns some combinatorial and decision problems on the growth function of formal languages. Given a language  $L$ , the *counting function*  $f_L$  of  $L$  is the map which associates with any non-negative integer  $n$ , the number  $f_L(n)$  of the words in  $L$  having length equal to  $n$ . The *growth function*  $g_L$  returns, for any non-negative integer  $n$ , the number of the words in  $L$  whose length is less than or equal to  $n$ . These two notions are interesting because the studying of the asymptotic behaviour of these functions gives some interesting informations on the structure of the language. In the theory of variable length codes [2], some important theorems utilize this notion: for instance, the well-known Kraft-Mc Millan-Slizard’s Inequality which gives a necessary condition for a set of words to be a code, or the Kraft’s theorem on the existence of a prefix code having a given counting function. In the theory of context-free languages, some remarkable results on the ambiguity of languages, based upon the counting function, are shown in [9]. Recently these notions have been the object of a quite active research, in particular with respect to languages of polynomial

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