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A NOTE ON ČERNÝ CONJECTURE FOR AUTOMATA OVER 3-LETTER ALPHABET

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ABSTRACT

We show that the alphabet size can play an essential role in the issue of automata synchronization. We give an example of 5-state automaton (over 3-letter alphabet) not isomorphic to Černý's one, with the minimal synchronizing word of length $(n-1)^2 = 16$. It is known [2] that there is no such automaton for 2-letter alphabet.

Keywords: Finite automata, synchronization, Černý Conjecture

1. Introduction

Let $\mathcal{A} = (Q, A, \delta)$ be an n-state deterministic finite automaton over an alphabet A, without initial and final states. We say that the word $w \in A^*$ synchronizes $\mathcal{A}$, if $|\delta(Q, w)| = 1$. Such automata are called synchronizing and w is called a synchronizing word for $\mathcal{A}$. If w is the shortest synchronizing word for $\mathcal{A}$ it is called the minimal synchronizing word. The Černý Conjecture [1] says that the length of the minimal synchronizing word in an n-state automaton does not exceed $(n-1)^2$. The conjecture was shown to be true for certain classes of automata (see [3, 4, 6]), but in general case it is still an open problem.

It is a well-known fact that for each $n \geq 2$ there exists an n-state automaton for which the conjectured upper bound $(n-1)^2$ is obtained. These are so-called "Černý automata" (see [1]), denoted here by $\mathcal{C}_n$. The examples of such "slowly synchronizable" automata over binary alphabet are extremely rare; beyond the series $\mathcal{C}_n$ we know only a few other examples. It is easy to find them for $n = 2, 3$. For $n = 4$ the example was given by Černý himself in [2]. There is no such automaton for $n = 5$. For $n = 6$ the example was given by J. Kari [5], as the counterexample to the stronger version of the Černý Conjecture proposed by J.-E. Pin [7, 8]. The examples given above exhaust our list of the known "slowly synchronizable" automata.

2. Automaton $\mathcal{A}_5$

Figure 1 shows a 5-state automaton $\mathcal{A}_5$ over 3-letter alphabet. It is easy to observe that neither of the three automata obtained from $\mathcal{A}_5$ by omitting one of the letters