Journal of Automata, Languages and Combinatorics **14** (2009) 1, 107–124

NONDETERMINISTIC ONE-TAPE OFF-LINE TURING MACHINES AND THEIR TIME COMPLEXITY [1]

GIOVANNI PIGHIZZINI
Dipartimento di Informatica e Comunicazione, Università degli Studi di Milano
via Comelico 39, I-20135 Milano, Italy
e-mail: pighizzini@dico.unimi.it

ABSTRACT

In this paper we consider the time and the crossing sequence complexities of one-tape off-line Turing machines. We show that the running time of each nondeterministic machine accepting a nonregular language must grow at least as $n \log n$, in the case all accepting computations are considered (accept measure). We also prove that the maximal length of the crossing sequences used in accepting computations must grow at least as $\log n$. On the other hand, it is known that if the time is measured considering, for each accepted string, only the faster accepting computation (weak measure), then there exist nonregular languages accepted in linear time. We prove that under this measure, each accepting computation should exhibit a crossing sequence of length at least $\log \log n$. We also present efficient implementations of algorithms accepting some unary nonregular languages.

Keywords: Turing machine, space complexity, time complexity, crossing sequence, unary language

1. Introduction

One of the main problems in the design of computer algorithms and in their implementation is that of producing efficient programs, under the restrictions given by the available resources.

For example, up to the first '80 years, when the central memories of the computers were very small and the operating systems of personal computers did not provide virtual memory functionalities, one of the critical topics in the implementation of applications using significative amounts of data was that of choosing data structures requiring only a small amount of extra memory, besides the memory needed by the actual data represented. Many times, computer programmers, in order to cope with a restricted space availability, were forced to choose data structures not efficient from the point of view of the time used by the algorithms manipulating them.

[1] Partially supported by MIUR under the project PRIN "Aspetti matematici e applicazioni emergenti degli automi e dei linguaggi formali: metodi probabilistici e combinatori in ambito di linguaggi formali".