

## LANGUAGES RECOGNIZED BY FINITE SUPERSOLUBLE GROUPS<sup>1</sup>

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### ABSTRACT

In this paper, we give two descriptions of the languages recognized by finite supersoluble groups. We first show that such a language belongs to the Boolean algebra generated by the modular products of elementary commutative languages. An elementary commutative language is defined by a condition specifying the number of occurrences of each letter in its words, modulo some fixed integer. Our second characterization makes use of counting functions computed by transducers in strict triangular form.

*Keywords:* Regular language, supersoluble group, variety of languages, representation theory

Eilenberg's variety theorem [7] is a powerful tool for classifying regular languages. It states that, given a variety of finite monoids  $\mathbf{V}$ , the class of languages  $\mathcal{V}$  whose syntactic monoid belongs to  $\mathbf{V}$  is a *variety of languages*, that is, a class of regular languages closed under finite union, complement, left and right quotients and inverse of morphisms. Further, the correspondence  $\mathbf{V} \rightarrow \mathcal{V}$  between varieties of finite monoids and varieties of languages is one-to-one and onto.

Eilenberg's theorem can be used in both ways: given a variety of languages, one can look for the corresponding variety of monoids, or, given a variety of monoids, one can seek for a combinatorial description of the corresponding variety of languages.

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<sup>1</sup>The authors acknowledge support from the AutoMathA programme of the European Science Foundation. The second author was supported by the Grant AINV07/093 from the Conselleria d'Empresa, Universitat i Ciència de la Generalitat Valenciana and the third author was supported by the Grant PR2007-0164 from MEC of Spain.