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LANGUAGES GENERATED BY CONTEXT-FREE GRAMMARS EXTENDED BY TYPE $AB \to BA$ RULES

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ABSTRACT

Derivations using branch-interchanging and language family obtained by context-free and interchange ($AB \to BA$) rules are analyzed. This language family is between the context-free and context-sensitive families. Closure properties and other properties are detailed. Only semi-linear languages can be generated in this way. Relation to partial commutations is shown.

Keywords: Formal languages, Chomsky hierarchy, derivation tree, interchange (permutation) rule, semi-linear languages, mildly context-sensitivity, partial commutations

1. Introduction

The Chomsky type grammars and the generated language families are one of the most basic and most important fields of theoretical computer science. The field is fairly old, the basic concept and results are from the middle of the last century (see, for instance, [12, 19, 21]). The context-free grammars (and languages) are widely used due to their generating power and simple way of derivation. The derivation trees represent the context-free derivations. It is well known that there is a big gap between the efficiency of context-free and context-sensitive grammars. There are very simple non-context-free languages as well, for instance $\{a^{2^n} \mid n \in \mathbb{N}\}$, $\{a^n b^n c^n \mid n \in \mathbb{N}\}$, etc. So, context-free grammars are not enough to describe several phenomenon of the world (in [6] seven circumstances are shown where the context-free grammars are not enough), but the context-sensitive family is too large, in applications it is too strong and so too much. Therefore several branches of extensions of context-free grammars were introduced by controlling the derivations in another way: In matrix grammars the rules are in matrices. Every matrix contains a sequence of rules. When a matrix is applied on a sentential form, all of its rules must be applied in their order [5]. In regularly controlled derivations the rules are labelled and a regular language is given over this label set. The word obtained by the concatenation of the labels of the used rules of the derivation must be in the given regular language in every terminating derivation [11]. Another way to direct the derivation by restricting the set of applicable rules is the programmed grammar [20]. These grammars (without