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ON THE SHUFFLE AUTOMATON SIZE FOR WORDS [1, 2]

Franziska Biegler, Mark Daley
Department of Computer Science University of Western Ontario,
London, ON N6A 5B7, Canada
e-mail: {fbiegler,daley}@csd.uwo.ca

and

Ian McQuillan
Department of Computer Science University of Saskatchewan,
Saskatoon, SK S7N 5A9, Canada
e-mail: mcquillan@cs.usask.ca

ABSTRACT

We investigate the state size of DFAs accepting the shuffle of two words. We provide an infinite family of words u and v, such that the minimal DFA for $u \sqcup\!\sqcup v$ requires an exponential number of states as a function of their lengths. We also show some conditions for the words u and v which ensure a quadratic upper bound on the state size of $u \sqcup\!\sqcup v$. Moreover, switching only two letters within one of u or v is enough to trigger the change from quadratic to exponential.

Keywords: Shuffle, words, finite languages, finite automata, state complexity

1. Introduction

Since its introduction, the shuffle operation has been aggressively studied as a model of nondeterministic interleaving in both purely theoretical and practical contexts. Perhaps due to the intrinsic nondeterminism of the operation, many problems concerning shuffle remain unsolved; e. g., shuffle decomposition for regular languages (though it is decidable [3] for commutative regular languages or locally testable languages while for context-free languages it is undecidable [3]).

We follow here the recent trend of attacking the special case of the shuffle of two words, inspired by attempts to solve the decomposition problem. It has been shown in [1] that shuffle decomposition on individual words is unique as long as there are two letters used within the words. In [2], the result from [1] was extended to show that if

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