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ON MEASURING NON-RECURSIVE TRADE-OFFS [1]

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ABSTRACT

We investigate the phenomenon of non-recursive trade-offs between descriptional systems in an abstract fashion. We aim at categorizing non-recursive trade-offs by bounds on their growth rate, and show how to deduce such bounds in general. We also identify criteria which, in the spirit of abstract language theory, allow us to deduce non-recursive tradeoffs from effective closure properties of language families on the one hand, and differences in the decidability status of basic decision problems on the other. We develop a qualitative classification of non-recursive trade-offs in order to obtain a better understanding of this very fundamental behaviour of descriptional systems.

Keywords: Descriptional systems; non-recursive trade-offs; level of unsolvability; arithmetic hierarchy

1. Introduction

In computer science in general, and also in the particular field of descriptional complexity, we try to classify problems and mechanisms according to different aspects of their tractability. Often the first distinction we make in such a classification is to check whether a problem admits an effective solution at all. If so, we usually take a closer look and analyze the inherent complexity of the problem. But undecidable problems can also be compared to each other, using the toolkit provided by computability theory. Here, it turns out that most naturally occurring problems are complete at some level of the arithmetic (or analytic) hierarchy. This has been a rather successful approach to understand the nature of many undecidable problems we encounter in various computational settings. As for decision problems, there are conversion problems between different models that cannot be solved effectively. Indeed, they evade solvability *a forteriori* because the size blow-up caused by such a conversion cannot be bounded above by any recursive function. This phenomenon, nowadays known as *non-recursive trade-off*, was first observed by Meyer and Fischer [20] between nondeterministic pushdown automata and finite automata. Previously, it had been known that every deterministic pushdown automaton accepting a regular language can be

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