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A COMPLETE DENSITY RANGE OF CONTINUUM-MANY NON-CONTEXT-FREE LANGUAGES FULFILLING STRONG ITERATION PROPERTIES

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ABSTRACT

We show that the cardinality of languages fulfilling all strong iteration lemmata for context-free languages, and having either an exponential, polynomial, or super-polynomial but sub-exponential density, is the cardinality of the continuum. In the last case we could prove this only for non-erasing iteration so far.

Keywords: Context-free languages, strong iteration lemmata, cardinality of languages

1. Introduction

In [1, 3, 4] it has been shown that there are continuously many languages fulfilling various iteration lemmata for context-free languages. However, languages with exponential density were given only. Exponential density is maximal. It is interesting to investigate also the cardinality of languages fulfilling iteration lemmata for context-free languages for polynomial and super-polynomial (but sub-exponential) densities. The question can be put also for polynomials of arbitrary but fixed degree k (there exist recursively enumerably many), as well as for arbitrary but fixed super-polynomial functions (there exist continuously many).

2. Definitions

Let $\mathbb{N} = \{0, 1, 2, \ldots\}$ denote the set of non-negative natural numbers. Let X be any alphabet, and $L \subseteq X^*$ a language over X. The *density* $\nu_L : X^* \longrightarrow \mathbb{N}$ is defined by