

GROUPS AND AUTOMATA: A PERFECT MATCH

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ABSTRACT

We present a personal perspective, inspired by our own research experience, of the interaction between group theory and automata theory: from Benois' Theorem to Stallings' automata, from hyperbolic to automatic groups, not forgetting the exotic automaton groups.

Keywords: automata, groups, free groups, hyperbolic groups, automatic groups, automaton groups

1. Introduction

Among abstract structures, it is groups which model the idea of symmetry in Mathematics. Moreover, the existence of inverses makes them a natural model for reversibility in theoretical computer science (see [37] for a model for *partial reversibility*). At the present time, when quantum computation takes its first steps (note that in quantum mechanics transformations are always assumed reversible), it is appropriate to make the history of the interaction between group theory and automata theory, undoubtedly the branch of theoretical computer science which has been playing the major role in the development of combinatorial and geometric group theory.

We intend this text to be a brief and light account of these interactions, under a personal perspective which emerged from our own work on the subject, and relating to our talk at DCFS 2012. We therefore chose to leave out finite groups (and the connections with group languages), being out of our own experience. Anyway, such connections are well known in theoretical computer science and can be easily found in the literature on finite automata [8, 32].

A deeper and more extended survey on the interactions groups/automata can be found out in two Handbook chapters written by Bartholdi and the author [4, 5].

We shall pay special attention to free groups: we introduce them in Section 2, discuss language-theoretic concepts in Section 3 and the representation of finitely generated subgroups by automata in Section 4. We shall also explain the role played by automata in the study of three important classes of groups: hyperbolic groups in Section 5, automatic groups in Section 6 and automaton groups (also known as