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ITERATING INVERSE BINARY TRANSDUCERS

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ABSTRACT

We study iterated transductions defined by a class of inverse transducers over the binary alphabet. The transduction semigroups of these automata turn out to be free Abelian groups and the orbits of finite words can be described as affine subspaces in a suitable geometry defined by the generators of these groups. We show that iterated transductions are rational for a subclass of our automata.

Keywords: Inverse transducers, transduction group, iteration, rationality.

1. Motivation

An *inverse transducer* is a type of Mealy automaton where all transitions are of the form $p \xrightarrow{a/\pi_p(a)} q$; here π_p is a permutation of the alphabet depending on the source state p. We only consider $\mathbf{2} = \{0, 1\}$ as input and output alphabet. Selecting an arbitrary state p as the initial state, we obtain a transduction $\mathcal{A}(p)$ from $\mathbf{2}^*$ to $\mathbf{2}^*$. These transductions can be viewed as automorphisms of the complete binary tree $\mathbf{2}^*$ and the collection of all transductions generates a subsemigroup $\mathcal{S}(\mathcal{A})$ of the full automorphism group $\mathsf{Aut}(\mathbf{2}^*)$. Similarly one can associate a group $\mathcal{G}(\mathcal{A})$ with $\mathcal{A}$ by including the inverses of all transductions. These groups are called *automata groups* or *self-similar groups* and have been studied in great detail in group theory and symbolic dynamics, see [11, 17] for extensive pointers to the literature. Automata groups have many interesting properties and have lead to elegant solutions to several outstanding problems. For example, Grigorchuk's well-known example of a group of intermediate growth has a description in terms of a 5-state inverse transducer.

[1]This work was done at Carnegie Mellon University.