δ-NEIGHBORHOOD CHROMATIC INDEX OF A GRAPH

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ABSTRACT

For any connected graph G of order $n \geq 3$, the δ-neighborhood chromatic index $\chi'_{\delta c}(G)$ is the minimum number of colors permitted in a proper edge coloring of the edges of a graph G such that for each edge e, e is adjacent to at least δ color classes.

In this paper we study the δ-neighborhood chromatic index of connected graphs of order $n \geq 3$ and obtain some bounds besides finding the exact value of this parameter for some graphs.

Keywords: δ-neighborhood coloring, dynamic coloring

1. Introduction

In this paper we deal with simple graphs G of *order* $n > 1$ (the number of vertices) and *size* $m > 1$ (the number of edges). We denote its edge set by $E(G)$ and its vertex set by $V(G)$.

Definitions and notation. We use the following definitions and notation from [5].

- $\delta(G)$ or δ, *minimum degree*: The degree of a vertex v in graph G, denoted deg v, is the number of edges incident with v. The minimum degree among the vertices of G is denoted $\delta(G)$ or δ.

- $\Delta(G)$ or Δ, *maximum degree*: The maximum degree among the vertices of G is denoted $\Delta(G)$ or Δ.

- $d(G)$, *diameter*: Let u and v be any two vertices in G. A shortest $u - v$ path is often called a *geodesic*. The diameter d(G) of a connected graph G is the length of any longest geodesic.

- $\chi(G)$, *chromatic number*: A *proper vertex coloring* of a graph G is an assignment of colors to the vertices of G (that is, $c : V(G) \to \mathcal{C}$, where $\mathcal{C}$ is a set of colors), such that no two adjacent vertices have the same color that is $c(x) \neq c(y)$ for every edge xy of G. The chromatic number $\chi(G)$ of G is the minimum number of colors permitted in such a coloring.