

LANGUAGES THAT REQUIRE FULL SCANNING OF WORDS TO DETERMINE MEMBERSHIP

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ABSTRACT

We introduce the *full scan condition* on a language, which captures the idea that the membership question for a word w cannot be determined from a given prefix, suffix pair. We study the corresponding one-sided conditions, establish relationships between them, characterize these languages in the regular case through their minimal automata and syntactic monoids, and develop techniques for testing regularity of a language. Finally we investigate a topology on the free monoid that arises in the course of our research.

Keywords: automata, regular languages, syntactic monoid, word scanning

1. Introduction

We investigate languages that require a complete scan of a given word, in various senses, in order to decide membership. The general concept represents the antithesis of the notion of a language where membership is determined by a set of allowable prefixes and suffixes. When these permitted one-sided factors form a finite set, the collection of such languages comprise a variety of regular languages the syntactic monoids of which are defined by the condition that all local submonoids are trivial ($eSe = e$ for all idempotents $e \in S$) ([10], also Theorem 3.6 of [12]). In contrast, the collection of full scan languages as we shall call them do not form a variety. However we generate families of examples and find that the class is closed under the adjoining or the deletion of so called chameleon sets. This latter class of languages form the closed sets of a T_1 topology on the free monoid. We also characterise the regular full scan languages in terms of properties of quotients, and of their minimal automata and their syntactic monoids, which yields useful tests for regularity. In this opening