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ON GRAPHS WITH REPRESENTATION NUMBER 3

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ABSTRACT

A graph $G = (V, E)$ is word-representable if there exists a word w over the alphabet V such that letters x and y alternate in w if and only if (x, y) is an edge in E. A graph is word-representable if and only if it is k-word-representable for some k, that is, if there exists a word containing k copies of each letter that represents the graph. Also, being k-word-representable implies being $(k + 1)$-word-representable. The minimum k such that a word-representable graph is k-word-representable, is called graph's representation number.

Graphs with representation number 1 are complete graphs, while graphs with representation number 2 are circle graphs. The only fact known before this paper on the class of graphs with representation number 3, denoted by $\mathcal{R}_3$, is that the Petersen graph and triangular prism belong to this class. In this paper, we show that any prism belongs to $\mathcal{R}_3$, and that two particular operations of extending graphs preserve the property of being in $\mathcal{R}_3$. Further, we show that $\mathcal{R}_3$ is not included in a class of c-colorable graphs for a constant c. To this end, we extend three known results related to operations on graphs.

We also show that ladder graphs used in the study of prisms are 2-word-representable, and thus each ladder graph is a circle graph. Finally, we discuss k-word-representing comparability graphs via consideration of crown graphs, where we state some problems for further research.

Keywords: word-representable graph, representation number, prism, ladder graph, circle graph, crown graph, comparability graph

1. Introduction

A graph $G = (V, E)$ is word-representable if there exists a word w over the alphabet $V = V(G)$ such that letters x and y alternate in w if and only if (x, y) is an edge in $E = E(G)$. It follows from definitions that word-representable graphs are a *hereditary class* of graphs. A comprehensive introduction to the theory of word-representable graphs is given in [3].

A graph is word-representable if and only if it is k-word-representable for some k, that is, if there exists a word containing k copies of each letter that represents the graph (see Theorem 3). By Proposition 4, being k-word-representable implies being $(k + 1)$-word-representable. The minimum k such that a word-representable graph G is k-word-representable, is called *graph's representation number*. This number is