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RECOGNIZING RELATIONS BY TREE AUTOMATA

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ABSTRACT

We introduce the relation $\xi(\mathbf{A})$ recognized by a tree automaton $\mathbf{A}$. For any tree automaton $\mathbf{A}$, it is decidable whether there is a deterministic tree automaton $\mathbf{B}$ such that $\xi(\mathbf{A}) = \xi(\mathbf{B})$. If the answer is yes, then we can construct such $\mathbf{B}$.

Keywords: recognizable relations; tree automata; formal languages

1. Introduction

We introduce a new way of recognizing relations over the set of ground terms analogously to the way tree automata recognize tree languages. We consider a tree automaton $\mathbf{A}$ with a state set A over a ranked alphabet Σ (denoted by $\mathbf{A}\langle A, \Sigma\rangle$). $\mathbf{A}$ recognizes the relation $\xi(\mathbf{A})$ consisting of all pairs (s, t) of ground trees such that $\mathbf{A}$ evaluates s and t to the same state. The class of relations recognized by tree automata is a proper subclass of $REC_\times$. Here $REC_\times$ is the class of all relations $\bigcup_{i=0}^{n} L_i \times M_i$, where $n \geq 0$, L_i, M_i are recognizable tree languages for $1 \leq i \leq n$, see Section 3.2.1 in [3]. Congruences of finite index over the term algebra are the same as the relations recognized by total deterministic tree automata. The recognizing capabilities of deterministic tree automata is strictly less than that of tree automata. For any tree automata $\mathbf{A}\langle A, \Sigma\rangle$ and $\mathbf{B}\langle B, \Sigma\rangle$, we can decide whether $\xi(\mathbf{A}) \subseteq \xi(\mathbf{B})$.

In the light of the definition of $REC_\times$, we study the union and intersection of $n \geq 0$ relations recognized by tree automata. For any tree automata $\mathbf{A}_1, \ldots, \mathbf{A}_n$, $n \geq 0$, over Σ, we can construct tree automata $\mathbf{B}$ and $\mathbf{C}$ such that $\bigcup_{i=1}^{n} \xi(\mathbf{A}_i) = \xi(\mathbf{B})$ and $\bigcap_{i=1}^{n} \xi(\mathbf{A}_i) = \xi(\mathbf{C})$. We give a tree automaton $\mathbf{A}$ such that there are no deterministic tree automata $\mathbf{A}_1, \ldots, \mathbf{A}_n$, $n \geq 0$, with $\xi(\mathbf{A}) = \bigcup_{i=1}^{n} \xi(\mathbf{A}_i)$.

We adopt a congruence relation $\rho \subseteq A \times A$ on a tree automaton $\mathbf{A}\langle A, \Sigma\rangle$ from [2]. It is called the determiner of $\mathbf{A}$. The quotient tree automaton $\mathbf{A}/\rho\langle A/\rho, \Sigma\rangle$ is called

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