

# NON-SELF-EMBEDDING LINEAR CONTEXT-FREE TREE GRAMMARS GENERATE REGULAR TREE LANGUAGES

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## ABSTRACT

For the class of linear context-free tree grammars, we define a decidable property called self-embedding. We prove that each non-self-embedding grammar in this class generates a regular tree language and show how to construct the equivalent regular tree grammar.

*Keywords:* context-free tree grammar, regular tree grammar, self-embedding, natural language processing

## 1. Introduction

In natural language processing (NLP), formal string grammars are used to approximate the set of all syntactically valid sentences of a language. Two important and successful grammar classes are the regular grammars (REGs) and the context-free grammars (CFGs) [16]. For these two classes there is a clear trade-off between expressive power and cost of processing, e. g., for parsing. It is undecidable whether an arbitrary given CFG generates a regular language [13, Thm. 8.15], but one may approximate a given context-free language by a REG, for example, in order to achieve better parsing complexity [22]. Alternatively, one may restrict CFGs to satisfy a decidable property that guarantees that they generate regular languages. Chomsky [3] defined such a property called non-self-embedding. A CFG is self-embedding if there are a nonterminal  $A$  and non-empty strings  $v$  and  $w$  over terminals and nonterminals such that  $A \Rightarrow^* vAw$ . He proved that each non-self-embedding CFG generates a regular language [3, Thm. 11]. In [22] self-embedding was expressed as a syntactic criterion, accompanied by a direct construction of a REG from a non-self-embedding CFG.

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