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UNRESTRICTED STATE COMPLEXITY OF BINARY OPERATIONS ON REGULAR AND IDEAL LANGUAGES

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ABSTRACT

We study the state complexity of binary operations on regular languages over different alphabets. It is known that if L'_m and L_n are languages of state complexities m and n, respectively, and restricted to the same alphabet, the state complexity of any binary boolean operation on L'_m and L_n is mn, and that of product (concatenation) is $m2^n - 2^{n-1}$. In contrast to this, we show that if L'_m and L_n are over different alphabets, the state complexity of union and symmetric difference is $(m+1)(n+1)$, that of difference is $mn+m$, that of intersection is mn, and that of product is $m2^n+2^{n-1}$. We also study unrestricted complexity of binary operations in the classes of regular right, left, and two-sided ideals, and derive tight upper bounds. The bounds for product of the unrestricted cases (with the bounds for the restricted cases in parentheses) are as follows: right ideals $m+2^{n-2}+2^{n-1}+1$ ($m+2^{n-2}$); left ideals $mn+m+n$ ($m+n-1$); two-sided ideals $m+2n$ ($m+n-1$). The state complexities of boolean operations on all three types of ideals are the same as those of arbitrary regular languages, whereas that is not the case if the alphabets of the arguments are the same. Finally, we update the known results about most complex regular, right-ideal, left-ideal, and two-sided-ideal languages to include the unrestricted cases.

Keywords: boolean operation, concatenation, different alphabets, left ideal, most complex language, product, quotient complexity, regular language, right ideal, state complexity, stream, two-sided ideal, unrestricted complexity

1. Motivation

Formal definitions are postponed until Section 2.

The first comprehensive paper on state complexity was published in 1970 by A. N. Maslov [21, p. 1373], but this work was unknown in the West for many years. Maslov wrote:

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