Journal of Automata, Languages and Combinatorics **22** (2017) 1–3, 123 – 143

THE COMPLEXITY OF LANGUAGES RESULTING FROM THE CONCATENATION OPERATION

Galina Jirásková[(A,B)] Alexander Szabari[(C,D)] Juraj Šebej[(C,D)]

[(A)] *Mathematical Institute, Slovak Academy of Sciences*
Grešákova 6, 040 01 Košice, Slovakia
`jiraskov@saske.sk`

[(C)] *Institute of Computer Science, Faculty of Science, P.J. Šafárik University*
Jesenná 5, 040 01 Košice, Slovakia
`alexander.szabari@gmail.com` `juraj.sebej@gmail.com`

ABSTRACT

We prove that for all m, n, and α with $1 \leq \alpha \leq f(m,n)$, where $f(m,n)$ is the state complexity of the concatenation operation, there exist a minimal m-state deterministic finite automaton A and a minimal n-state deterministic finite automaton B, both defined over an alphabet Σ with $|\Sigma| \leq 2n+4$, such that the minimal deterministic finite automaton for the language $L(A)L(B)$ has exactly α states. This improves a similar result in the literature that uses an exponential alphabet.

Keywords: regular languages, deterministic finite automata, concatenation, state complexity, the magic number problem

1. Introduction

In 2000, Iwama et al. [3] stated the question of whether there always exists a minimal nondeterministic finite automaton (NFA) of n states whose equivalent minimal deterministic finite automaton (DFA) has α states for all integers n and α satisfying $n \leqslant \alpha \leqslant 2^n$. The question was also considered by Iwama et al. [4], and answered positively in [7] for a ternary alphabet. However, in the unary case, the existence of holes, so called "magic numbers" was proved by Geffert [1]. The binary case is still open.

The same problem on subregular language families was studied by Holzer et al. [2]. It turned out that the existence of non-trivial magic numbers is rare, and that the ranges of possible complexities are usually contiguous. One interesting exception was obtained by Čevorová [14]. She studied the star operation on unary languages, and proved that there are two linear segments of magic numbers in the range from 1 to

[(B)] Research supported by grant VEGA 2/0084/15 and grant APVV-15-0091.
[(D)] Research supported by grant VEGA 1/0142/15 and grant APVV-15-0091.