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THE DESCENT STATISTIC ON SIGNED SIMSUN PERMUTATIONS

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ABSTRACT

In this paper, we study the generating polynomials obtained by enumerating signed and even-signed simsun permutations by number of descents. Properties of the polynomials, including the recurrence relations and generating functions are studied.

Keywords: signed simsun permutations, even-signed simsun permutations, descents

1. Introduction

Let $\mathfrak{S}_n$ denote the symmetric group of all permutations of $[n]$, where

$$[n] = \{1, 2, \ldots, n\}.$$

Let $\pi = \pi(1)\pi(2)\cdots\pi(n) \in \mathfrak{S}_n$. A *descent* in π is an index i such that $\pi(i) > \pi(i+1)$, where $i \in [n-1]$. We say that $\pi \in \mathfrak{S}_n$ has no *double descents* if there is no index $i \in [n-2]$ such that $\pi(i) > \pi(i+1) > \pi(i+2)$. A permutation $\pi \in \mathfrak{S}_n$ is called *simsun* if for all k, the subword of π restricted to $[k]$ (in the order they appear in π) contains no double descents. For example, 35142 is simsun, but 35241 is not (if we restrict the permutation 35241 to [3], we get 321 which contains a double descent). Let $\mathcal{RS}_n$ be the set of simsun permutations of length n. Let $|C|$ denote the cardinality of a set C. Simion and Sundaram [15, p. 267] discovered that $|\mathcal{RS}_n| = E_{n+1}$, where E_n is the nth Euler number (see [14] for instance).

There have been extensive studies of the descent polynomials for simsun permutations (see [5, 12, 15] for instance). Let

$$\operatorname{des}_A(\pi) = |\{i \in [n-1] : \pi(i) > \pi(i+1)\}|,$$