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## PROPERTIES OF LANGUAGE CLASSES BETWEEN LINEAR AND CONTEXT-FREE

Henning  $\operatorname{Fernau}^{(A)}$  Lakshmanan  $\operatorname{Kuppusamy}^{(B)}$  Indhumathi  $\operatorname{Raman}^{(C)}$ 

(A) Fachbereich 4 - CIRT, Universität Trier 54286 Trier, Germany fernau@uni-trier.de (H. FERNAU)

<sup>(B)</sup>School of Computer Science and Engineering, VIT Vellore-632 014, India klakshma@vit.ac.in (L. KUPPUSAMY)

(C) School of Information Technology and Engineering, VIT Vellore-632 014, India indhumathi.r@vit.ac.in (I. RAMAN)

## ABSTRACT

We study several aspects of language classes between linear and context-free, namely, hierarchical aspects, closure properties and normal forms. We show how to apply such normal form results to matrix grammars with quite restricted nonterminal complexity.

 $K\!eywords:$  linear languages, meta-linear languages, rational closure, normal forms, language hierarchies, closure properties, matrix grammars

## 1. Introduction

The linear languages LIN are one of the best-known classes of languages that lie strictly between the regular languages REG and the context-free languages CF, i. e., they form a specific intermediate layer within the Chomsky hierarchy. Linear languages are mentioned in most introductory textbooks on Formal Languages. They enjoy many (but not all) of the nice algebraic properties of REG and CF. More precisely, LIN forms a so-called semi-AFL, i. e., it is closed under the trio operations (or equivalently, under rational transductions) and under union, but unlike REG or CF, it does not form an abstract family of languages (AFL), because it is not closed under concatenation or Kleene star in addition.

This makes it interesting to study language classes that lie between LIN and CF and enjoy more (or other) closure properties; a rather well-known language class of this type is the class MLIN comprising of the meta-linear languages. This has been already nicely done by Kutrib and Malcher in [20] (also cf. [27, Sec. 6]). This paper supplements that study (i) by focussing on language classes that are only slightly