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A CLASS OF RATIONAL RELATIONS GENERALISING THE SUBWORD ORDER

Matthew McDevitt[(A)]

School of Mathematics and Statistics, University of St Andrews
Mathematical Institute, North Haugh, St Andrews KY16 9SS, Scotland
mm241@st-andrews.ac.uk

ABSTRACT

Using transducers, we introduce a new class of relations on words which generalise the subword order. We show it is decidable whether a given relation in this class is an order. For certain orders in this class we show that the property of being a well-quasi-order is decidable.

Keywords: words, transducers, rational relations, order, well-quasi-order

1. Introduction

The purpose of this paper is to introduce a family of orders on the free monoid A^* generated by certain transducers, and discuss the decidability of their properties, with a particular focus on deciding the property of being a well-quasi-order (WQO).

Orderings of words over a finite alphabet are of significance throughout mathematics and theoretical computer science. For instance, the celebrated Higman's Theorem [8], asserting that A^* is WQO under the subword (subsequence) ordering, is a ubiquitous tool in combinatorics. This theorem can be used to show that certain combinatorial classes are WQO, via an encoding of the objects in these classes to words. Likewise, the fact that the factor (contiguous subsequence) ordering is *not* WQO is used to demonstrate that various combinatorial classes fail to be WQO, by encoding the basic antichain $\{ ab^n a \mid n \in \mathbb{N} \}$. For more details and references see the recent survey article [10]. Another application of Higman's Theorem is in proving a result of Haines [7], asserting that the downward closure of *any* language with respect to the subword ordering is regular. By abstracting a language by its (regular) downward closure, certain problems become decidable; see the recent work of Zetzsche [14] for more details. Higman's theorem is also used to prove central results about well-structured transition systems [5] and lossy channel systems [3]. To give another famous example of the importance of WQOs on words, Ehrenfeucht et al. [4] characterise regular languages as precisely the upward closed sets under WQOs

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