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EMBEDDING RATIONALLY INDEPENDENT LANGUAGES INTO MAXIMAL ONES

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ABSTRACT

We consider the embedding problem in coding theory: given an independence (a code-related property) and an independent language L, find a maximal independent language containing L. We solve the problem by providing an embedding formula for the case where the code-related property is defined via a rational binary relation that is decreasing with respect to any fixed total order on the set of words. Our method works by iterating a max-min operator that has been used before for the embedding problem for properties defined by length-increasing-and-transitive binary relations. We use input-decreasing transducers to represent order-decreasing rational relations, and we show that these transducers can describe many known properties from both the noiseless and noisy domains of coding theory, as well as any combination of such properties. Moreover, in many cases the desired maximal embedding is effectively computable. Finally, we show that there is no embedding formula when the given code-related property is defined via an input-altering transducer.

Keywords: languages, maximal codes, embedding, variable-length codes, transducers, error control codes

1. Introduction

The embedding problem for a language L satisfying a property $\mathcal{P}$ is to find a language L' that contains L and is maximal satisfying $\mathcal{P}$. This problem is meaningful when the property $\mathcal{P}$ is an independence. In particular, many natural code-related properties are independences with respect to binary relations on words. In this setting, a binary relation ρ defines the property that consists of all languages in which no two different words are related via ρ. Such languages are called ρ-independent. The embedding problem has been addressed well for properties defined by length-increasing-and-transitive relations [24], as well as for several fixed properties like the bifix code property [28], the solid code property [14], and the bounded deciphering delay property [3]. In [6], the authors consider properties where the relation ρ is

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