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SYNCHRONIZING NON-DETERMINISTIC FINITE AUTOMATA

Henk Don[(A)] Hans Zantema[(B,A)]

[(A)] *Radboud University Nijmegen*
P.O. Box 9010, 6500 GL Nijmegen, The Netherlands
`h.don@math.ru.nl`

[(B)] *Department of Computer Science, TU Eindhoven*
P.O. Box 513, 5600 MB Eindhoven, The Netherlands
`h.zantema@tue.nl`

ABSTRACT

In this paper, we show that every D3-directing CNFA can be mapped uniquely to a DFA with the same synchronizing word length. This implies that Černý's conjecture generalizes to CNFAs and that any upper bound for the synchronizing word length of DFAs is an upper bound for the D3-directing word length of CNFAs as well. As a second consequence, for several classes of CNFAs sharper bounds are established. Finally, our results allow us to detect all critical CNFAs on at most 6 states. It turns out that only very few critical CNFAs exist.

Keywords: Černý conjecture, non-deterministic finite automata, D3-directing

1. Introduction and Preliminaries

In this paper we study synchronization of non-deterministic finite automata (NFAs). As is the case for deterministic finite automata (DFAs), symbols define functions on the state set Q. However, in an NFA symbols are allowed to send a state to a subset of Q, rather than to a single state. An NFA is called complete if these subsets are non-empty. This basically says that in every state, every symbol has at least one outgoing edge. Formally, a *complete non-deterministic finite automaton (CNFA)* $\mathcal{A}$ over a finite alphabet Σ consists of a finite set Q of states and a map $\delta : Q \times \Sigma \to 2^Q \setminus \{\emptyset\}$. We denote the number of states by $|\mathcal{A}|$ or by $|Q|$.

A DFA is called synchronizing if there exists a word that sends every state to the same fixed state. In 1964, Černý [4] conjectured that a synchronizing DFA on n states always admits a synchronizing (or directing, reset) word of length at most $(n-1)^2$. He gave a sequence C_n of DFAs in which the shortest synchronizing word attains this bound. In this paper, we denote the maximal length of a shortest synchronizing word