Journal of Automata, Languages and Combinatorics **24** (2019) 1, 3–12
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THE k-DIMENSIONAL CUBE IS k-REPRESENTABLE

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ABSTRACT

A graph is called k-representable if there exists a word w over the nodes of the graph, each node occurring exactly k times, such that there is an edge between two nodes x, y if and only after removing all letters distinct from x, y, from w, a word remains in which x, y alternate. We prove that if G is k-representable for $k > 1$, then the Cartesian product of G and the complete graph on n nodes is $(k + n - 1)$-representable. As a direct consequence, the k-dimensional cube is k-representable for every $k \geq 1$.

Our main technique consists of exploring occurrence-based functions that replace every ith occurrence of a symbol x in a word w by a string $h(x, i)$. The representing word we construct to achieve our main theorem is purely composed from concatenation and occurrence-based functions.

Keywords: k-dimensional cube, word representation, cartesian product graph

1. Introduction

For a word w over an alphabet A, two letters x and y are said to *alternate* in w if between every two x's in w a y occurs and between every two y's in w an x occurs. Stated otherwise: deleting all letters but x and y from w results in a word $xyxy\dots$ or $yxyx\dots$ of even or odd length.

A graph $G = (V, E)$ is defined to be *word-representable*, or shortly *representable*, if there is a word w over the alphabet V, such that $(x, y) \in E$ if and only if x and y alternate in w. The word w is said to *represent*, or be a *representant* of, G. A word only represents one graph, while a graph can have multiple words representing it.

A lot of work has been done on investigating which graphs are word-representable; this is the main topic of the book [7]. For a more recent overview see [6]. Related work includes [4, 2, 9, 3]. A first basic observation is that one may restrict oneself just to considering *uniform* words: a word w over an alphabet A is called *uniform* if there exists a number k such that every letter in A occurs exactly k times in w. For such k, the word w is called *k-uniform.* So the basic observation states