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A NEW LOWER BOUND FOR RESET THRESHOLD OF BINARY SYNCHRONIZING AUTOMATA WITH SINK

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ABSTRACT

We present a new series of examples of binary slowly synchronizing automata with a sink state. The reset threshold of the n-state automaton in this series is $\frac{n^2}{4} + 2n - 9$. This improves on the previously known lower bound for the maximum reset threshold of binary synchronizing n-state automata with a sink state.

Keywords: synchronizing 0-automaton, reset threshold

1. Background and Motivation

Let $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ be a complete deterministic finite automaton (DFA, for short) with the state set Q, the input alphabet Σ, and the transition function $\delta\colon Q \times \Sigma \to Q$. If $|\Sigma| = 2$, then we refer to this automaton as a *binary* DFA. The action of the letters in Σ on the states in Q defined via δ extends in a natural way to an action of the words in the free Σ-generated monoid Σ^*; the latter action is still denoted by δ. For any $w \in \Sigma^*$ and $X \subseteq Q$, we set

$$\delta(X, w) = \{\ \delta(q, w) \mid q \in X\ \}.$$

Sometimes we write $X.w$ for $\delta(X, w)$.

A DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is said to be *synchronizing* if there is a word $w \in \Sigma^*$ such that $|\delta(Q, w)| = 1$. The word w is then called a *synchronizing* or *reset* word for $\mathscr{A}$. The minimum length of reset words for a synchronizing automaton $\mathscr{A}$ is called the *reset threshold* of $\mathscr{A}$ and is denoted by $\mathrm{rt}(\mathscr{A})$. The reset threshold of a class of

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