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LOCALLY STRONGLY TRANSITIVE AUTOMATA IN THE ČERNÝ CONJECTURE AND RELATED PROBLEMS

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ABSTRACT

This paper mainly concerns the property of strong local transitivity of finite automata. We will survey some results on this property in the context of the synchronization problem. Further aspects and results relating this problem with the Road coloring problem of finite directed graphs and the Restivo conjecture on the minimal length of an incompletable word of a finite language, are also analyzed.

Keywords: Černý conjecture, synchronizing automaton, road coloring problem, incompletable word, synchronizing set, complete set

1. Introduction

The *synchronization problem* for a deterministic automaton consists in the search of an input-sequence, called a *synchronizing* or *reset word*, such that the state attained by the automaton, when this sequence is read, does not depend on the initial state of the automaton itself. If such a sequence exists, the automaton is called *synchronizing*.

Three problems which have been intensively investigated in the last decades are based upon this concept: the *Černý conjecture*, the *Road coloring problem*, and the *Restivo conjecture* on the minimal length of an incompletable word of a finite language.

The aim of this paper is to survey results obtained by the authors in the last ten years of research activity on these three problems. These results (see [14–21]) are based upon the notion of *locally strongly transitive automaton* introduced in [16].

The Černý conjecture formulated by the Slovak scientist J. Černý in 1964 is one of the most famous longstanding open problem in Automata Theory [22]. It claims that every deterministic synchronizing n-state automaton has a reset word of length $(n-1)^2$. In the last 55 years, this conjecture and some related problems have been intensively investigated, bringing to the discovery of important results on the combinatorial structure of these objects. The interested reader is refered to [47] for a