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ON RANDOM PRIMITIVE SETS, DIRECTABLE NFAs AND THE GENERATION OF SLOWLY SYNCHRONIZING DFAs

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ABSTRACT

We tackle the problem of the randomized generation of slowly synchronizing deterministic automata (DFAs) by generating random primitive sets of matrices. We show that when the randomized procedure is too simple the exponent of the generated sets is of order $O(n \log n)$ with high probability, thus the procedure fails to return DFAs with large reset threshold. We extend this result to random nondeterministic automata (NFAs) by showing, in particular, that a uniformly sampled NFA has both a 2-directing word and a 3-directing word of length $O(n \log n)$ with high probability. We then present a more involved randomized algorithm that manages to generate DFAs with large reset threshold and we finally leverage these findings for exhibiting new families of DFAs with reset threshold of order $\Omega(n^2/4)$.

Keywords: synchronizing automaton, random automaton, Černý conjecture, directing nondeterministic automaton, random matrix set, primitive set

1. Introduction

A complete deterministic finite state automaton (DFA) is *directing* or *synchronizing* if it admits a word that brings the automaton from every state to the same fixed state; a word of this kind is called a *directing* or *synchronizing* word. More formally, a DFA is a triple $\mathcal{A} = \langle Q, \Sigma, \delta \rangle$ where Q is a finite set of states, Σ is a finite set of input symbols called the alphabet and $\delta : Q \times \Sigma \to Q$ is the transition function. A synchronizing word w is a finite sequence of letters of Q for which there exists a vertex $v \in Q$ such that $\delta(q, w) = v$ for every $q \in Q$, where δ has been extended

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