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COUNTING SYMBOL SWITCHES IN SYNCHRONIZING AUTOMATA

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ABSTRACT

Instead of looking at the lengths of synchronizing words as in Černý's conjecture, we look at the *switch count* of such words, that is, we only count the switches from one letter to another. Where the synchronizing words of the Černý automata $\mathcal{C}_n$ have switch count linear in n, we wonder whether synchronizing automata exist for which every synchronizing word has quadratic switch count. The answer is positive: we prove that switch count has the same complexity as synchronizing word length. We give some series of synchronizing automata yielding quadratic switch count, the best one reaching $\frac{2}{3}n^2 + O(n)$ as switch count.

We investigate all binary automata on at most 9 states and determine the maximal possible switch count. For all $3 \leq n \leq 9$, a strictly higher switch count can be reached by allowing more symbols. This behaviour differs from length, where for every n, no automata are known with higher synchronization length than $\mathcal{C}_n$, which has only two symbols. It is not clear if this pattern extends to larger n. For $n \geq 12$, our best construction only has two symbols.

Keywords: Černý conjecture, synchronization, switch count

1. Introduction

The well-known Černý automaton $\mathcal{C}_n$ on n states has the shortest synchronizing word $b(a^{n-1}b)^{n-2}$ of length $(n-1)^2$; for $n = 4$ this is *baaabaaab* and the automaton is drawn below.