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HARDLY REACHABLE SUBSETS AND COMPLETELY REACHABLE AUTOMATA WITH 1-DEFICIENT WORDS

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ABSTRACT

This article focuses on subset reachability in synchronizing automata.

First, we study the length of shortest words reaching subsets of states in synchronizing automata. We provide an automata family with subsets that cannot be reached by words shorter than $2^n/n$, thus disproving a recent conjecture of Don. We then analyze relaxed versions of this conjecture.

Second, we analyze the Γ_1-graph construction. The Γ_1-graph is derived from 1-deficient words and is a key tool for studying completely reachable automata. We introduce the concept of roots of 1-deficient words, which allows to state explicit concatenation rules for these words. Based on these results, we provide a polynomial-time algorithm for constructing the Γ_1-graph. Then, we disprove a conjecture by Bondar and Volkov linking the strong connectivity of this graph and the concatenation of 1-deficient words of completely reachable automata. Finally, we prove an alternative version of this conjecture.

Keywords: Γ_1-graph, subset reachability, short reaching word, completely reachable automata

1. Introduction

Automata[1] are useful tools in various fields of applied mathematics. In pattern recognition, they allow to parse texts and efficiently find letter sequences. In language theory, they are the basic tools defining formal languages and context free languages. In theoretical computer science, automata provide simple models for the behaviour of computing devices. More recently, automata have also been at the core of synthesis and verification of complex automated systems. A general presentation of automata and their applications is provided by Berthé and Rigo [4], and by Linz [22].

Synchronization is an important topic in automata theory. Indeed, if a machine can be modelled as a synchronizing automaton, then it is possible to fix its state

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[1] Formal definitions are provided in the next subsection.