Journal of Automata, Languages and Combinatorics **24** (2019) 2–4, 343 – 365

THE ČERNÝ CONJECTURE HOLDS WITH HIGH PROBABILITY

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ABSTRACT

An automaton is synchronizing when there is a word that brings every state into one and the same state. Such a word is called a synchronizing word, and Černý conjectured in 1964 that if a n-state deterministic automaton is synchronizing, then it has a synchronizing word of length at most $(n-1)^2$. The best bound known so far is cubic in n and was obtained by Szykuła in 2017.

In this article, we study the synchronization properties of random deterministic automata, for the uniform distribution. Berlinkov recently proved that they are synchronizing with high probability. Our contribution is to study the typical length of the smallest synchronizing word, when such a word exists: we establish that with high probability, such an automaton with n states admits a synchronizing word of length $\mathcal{O}(n \log^3 n)$. As a byproduct, we get that for most automata, the Černý conjecture holds.

Keywords: random automata, synchronization, discrete probabilities

1. Introduction

For a given automaton, a *synchronizing word* (or a *reset word*) is a word that brings that automaton into one and the same state, regardless of the starting position. This notion, first formalized by Černý in the sixties, arises naturally in automata theory and its extensions, and plays an important role in several application areas, most of them related to the idea of being able to reset a device from every unknown state (see [30] for some examples). Beside applications, one of the reasons synchronizing automata are still intensively studied in theoretical computer science is the following question asked by Černý [9] back in 1964: "*Does every synchronizing n-state automaton admit a synchronizing word of length at most* $(n-1)^2$*?*" The upper bound of $(n-1)^2$, as shown by Černý by providing a matching family of automata, is best possible. This question, now known as *the Černý conjecture*, is one of the most famous conjectures in automata theory. Though established for important subclasses of automata, the Černý conjecture remains open in the general case. The best known bound is $\left(\frac{7}{48} + \frac{2\cdot 15\,625}{1\,597\,536}\right) n^3 + o(n^3)$ which is due to Shitov [27] who improved the