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AN IMPROVEMENT TO A RECENT UPPER BOUND FOR SYNCHRONIZING WORDS OF FINITE AUTOMATA

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ABSTRACT

It has been known since the 60's that every synchronizing complete discrete n-state automaton admits a reset word of length at most $\alpha n^3 + o(n^3)$ for some absolute constant α. J.-E. Pin and P. Frankl proved this statement with $\alpha = 1/6 = 0.1666\ldots$ in 1982, and this bound remained best known until 2017, when M. Szykuła decreased its value to $\alpha \approx 0.1664$. In this note, we present a modification to the latest approach which leads to a more substantial improvement of $\alpha \leq 0.1654$.

Keywords: automata theory, Černý conjecture

1. Introduction

Let $\mathcal{A} = (Q, \Sigma, \delta)$ be a *deterministic finite automaton*, where Q is a finite set of *states*, Σ is a finite *alphabet*, and $\delta : Q \times \Sigma \to Q$ is a *transition function*, which assigns a mapping $Q \to Q$ to every letter of Σ. This function naturally extends to an action $Q \times \Sigma^* \to Q$ of the free monoid Σ^* on Q, and this action is still denoted by δ. For a subset $S \subseteq Q$ and a word $w \in \Sigma^*$, we define $S \cdot w$ as the set of all images $s \cdot w$ of elements $s \in S$ under the action of w. The cardinality of $Q \cdot w$ is called the *rank* of a word w; this quantity is denoted by $\operatorname{rk}_{\mathcal{A}} w$ or simply $\operatorname{rk} w$ if the choice of $\mathcal{A}$ is clear from the context. The *rank of an automaton* is defined as the smallest possible rank of a word. An automaton $\mathcal{A}$ of rank one is called *synchronizing*, and the length of the shortest rank-one words is called the *reset threshold* of $\mathcal{A}$ and denoted by $\operatorname{rt}(\mathcal{A})$.

Upper bounds on reset thresholds of synchronizing automata were a topic of extensive research in the last 50 years, and one of the main goals of this study is a famous conjecture stating that $\operatorname{rt}(\mathcal{A}) \leq (n-1)^2$ for any synchronizing n-state automaton $\mathcal{A}$; this statement was considered many years ago by different authors and became known as the *Černý conjecture* (see a historical survey in [11]). There is a lot of progress on this question for different special classes of automata [5, 7, 9], but the general version of the Černý conjecture remains wide open. The cubic upper bounds on the reset threshold, that is, inequalities of the form $\operatorname{rt}(\mathcal{A}) \leq \alpha n^3 + o(n^3)$ for some fixed α, have been known since 1966, see [6]. After a series of improvements [1, 2, 4, 5], the