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SLOWLY SYNCHRONIZING AUTOMATA WITH IDEMPOTENT LETTERS OF LOW RANK

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ABSTRACT

We use a semigroup-theoretic construction by Peter Higgins in order to produce, for each even n, an n-state and 3-letter synchronizing automaton with the following two features:

1) all its input letters act as idempotent selfmaps of rank $\frac{n}{2}$;

2) its reset threshold is asymptotically equal to $\frac{n^2}{2}$.

Keywords: synchronizing automaton, reset threshold, rank of a letter, idempotent selfmap

1. Background and Overview

A *complete deterministic finite automaton* (DFA) is a triple $\langle Q, \Sigma, \delta\rangle$, where Q and Σ are finite sets called the *state set* and the *input alphabet* respectively, and $\delta\colon Q\times\Sigma \to Q$ is a totally defined map called the *transition function.* Let Σ^* stand for the collection of all finite words over the alphabet Σ, including the empty word. The transition function extends to a function $Q\times\Sigma^* \to Q$, still denoted δ, in the following natural way: for every $q \in Q$ and $w \in \Sigma^*$, we set $\delta(q, w) := q$ if w is empty and $\delta(q, w) := \delta(\delta(q, v), a)$ if $w = va$ for some $v \in \Sigma^*$ and some $a \in \Sigma$. Thus, every word $w \in \Sigma^*$ induces the selfmap $q \mapsto \delta(q, w)$ of the set Q; we say that w is *idempotent* if so is the selfmap induced by w, that is, if $\delta(q, w) = \delta(q, w^2)$ for each $q \in Q$.

When we deal with a fixed DFA, we simplify our notation by suppressing the sign of the transition function; this means that we introduce the DFA as a pair $\langle Q, \Sigma\rangle$ rather than a triple $\langle Q, \Sigma, \delta\rangle$ and write $q.w$ for $\delta(q, w)$ and $Q.w$ for $\{ \delta(q, w) \mid q \in Q \}$.

A DFA $\mathscr{A} = \langle Q, \Sigma\rangle$ is called *synchronizing* if there exists a word $w \in \Sigma^*$ whose action *resets* $\mathscr{A}$, that is, w leaves the automaton in one fixed state, regardless of the state at which w is applied. This means that $q.w = q'.w$ for all $q, q' \in Q$. Any word w

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