Journal of Automata, Languages and Combinatorics **26** (2021) 3–4, 221 – 253
© Institut für Informatik · Justus-Liebig-Universität Giessen

AUTOMATA, PALINDROMES, AND REVERSED SUBWORDS

Lukas Fleischer[(A)] Jeffrey Shallit[(A,B)]

[(A)] *School of Computer Science, University of Waterloo*
Waterloo, ON N2L 3G1, Canada
`lfleischer@lfos.de` `shallit@uwaterloo.ca`

ABSTRACT

In 2013, Fici and Zamboni proved a number of theorems about finite and infinite words having only a small number of factors that are palindromes. Earlier, in 2005, Rampersad and the second author had proved a number of theorems about infinite words $\mathbf{x}$ with the property that if w is any sufficiently long finite factor of $\mathbf{x}$, then its reversal w^R is *not* a factor of $\mathbf{x}$. In both cases, the arguments used were typically case-based and somewhat involved.

In this note we rederive most of these results, and obtain many new ones, by a different method based on finite automata. Two variations of the method are presented. One advantage to our method is that it replaces complicated case-based proofs with (relatively simple) machine computations. Another advantage is that our method can provide detailed enumeration results about the number of words satisfying the various conditions. We explore these ideas in detail.

Keywords: finite automaton, regular language, palindrome, reversed factor, pattern avoidance

1. Introduction

In this paper we are concerned with certain avoidance properties of finite and infinite words over a finite alphabet Σ. Our default alphabet is $\Sigma_k = \{0, 1, \ldots, k-1\}$ for an integer $k \geq 2$. For a language $L \subseteq \Sigma_k^*$, we write $\overline{L}$ for $\Sigma_k^* - L$, the complement of L.

Recall that a word x is said to be a *factor* of a word w if there exist words y, z such that $w = yxz$. For example, the word `act` is a factor of the English word `factor`. We sometimes say w *contains* x. (Another commonly-used term for factor is *subword*, although this latter term often refers to a different concept entirely.) We say a (finite or infinite) word x *avoids* a set S if no element of S is a factor of x.

The reverse of a finite word x is written x^R. Thus, for example,

$$(\texttt{drawer})^R = \texttt{reward}.$$

[(B)] Research of Jeffrey Shallit supported by NSERC grant 2018-04118.